



## Technical Note

## On the effective heat transfer parameters in a packed bed

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Received 20 January 1999

**1. Introduction**

Heat transfer behavior of fluid flow through packed tube and channel has an important impact on the design and operation of catalytic reactors. McGreavy et al. [1] emphasized how the accuracy of the heat transfer parameters can effect the predicted temperature distributions of reactors for the catalytic partial oxidation of benzene by utilizing two sets of correlations to predict effective heat transfer coefficient  $h_w$  and effective radial thermal conductivity  $k_{er}$ . These effective transport parameters reflect the increased thermal resistance in the vicinity of the wall. The ratio of tube-to-particle diameter  $d_t/d_p$  is directly related to the properties of the wall region especially for the gas-phase system due to increase in porosity [2–6]. When  $d_t/d_p$  is small, the variation of porosity and velocity may cover the significant part of the bed [2,4,5].

It has been difficult to obtain satisfactory correlations of heat transfer coefficients for packed beds with small values of  $d_t/d_p$  [1,5]. Dixon [4,5] studied the region  $5 < d_t/d_p < 12$ , and  $d_t/d_p < 4$ , while Borkink and Westerterp [6,7] studied the region  $7 < d_t/d_p < 27$ , and they determined  $Nu$  and  $k_{er}$  for various size and shape of packing and pointed out that the mixing phenomena on a particle scale may play an important role in the process of radial heat transfer. Tsotsas and Schlunder [8] associated the heat transfer coefficient with the spatial extent of the wall region with respect to tube diameter.

The procedure of Coberly and Marshall [9] for

determining the effective transport parameters from the spatial distribution of fluid temperature has been widely used [8]. For this purpose, three types of experimental procedures are utilized: (i) the wall heated packed bed heat transfer measurements without a reaction, (ii) radial heat transfer in exothermic catalytic reactors, (iii) and lastly a mass transfer controlled electrochemical reaction taking place on the wall of the tube [2,8,10]. Although, it has been a common practice to correlate  $h_w$  and  $k_{er}$  in terms of a Reynolds number based on the particle diameter  $d_p$ , there has been no general agreement on the form of the correlations for the packed beds. The wall Nusselt number ( $Nu = h_w d_p / k_f$ ) is expressed in a linear form [5,6,11,12], as well as in an exponential form [7,13–15] with respect to  $Re$  or the Peclet number  $Pe$ . The correlations of  $k_{er}/k_f$  are mainly expressed in a linear form with respect to  $Re$  or  $Pe$  [4,5,7,16]. The correlations for  $Nu$  show considerable scatter and discrepancies [8]. The definition of the fluid temperature in determining  $Nu$  is based on the bulk temperature of the fluid  $T_b$  [15,18–20] or is based on the fluid temperature at the wall  $T_{fw}$  [8,11,17]. Therefore, two approaches for the determination of  $Nu$  based on the temperature differences of  $(T_w - T_b)$  and  $(T_w - T_{fw})$  have appeared in the literature.

In this study, wall-to-fluid heat transfer coefficient  $h_w$  and effective radial thermal conductivity  $k_{er}$  have been determined for a packed bed with  $4.5 < d_t/d_p < 7.5$  in the region of  $200 < Re < 1450$ . Two values of  $Nu$  based on the bulk as well as the extrapolated fluid temperature at the wall have been determined and compared.

**2. Experimental**

Experiments were carried out in a rectangular duct

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**Nomenclature**

$a_p$	specific surface	$W$	width of the duct
$d$	diameter	$x$	axial direction
$h$	heat transfer coefficient	$y$	radial direction
$H$	separation distance between top and bottom walls		
$k$	thermal conductivity		<i>Greek symbols</i>
$K$	constant in Eq. (5)	$\varepsilon$	void fraction
$L$	packed section length of the duct	$\nu$	kinematic viscosity
$Nu, a$	Nusselt number based on the particle diameter, and $(T_w - T_{fw})$ , Eq. (1)	$\Phi$	shape factor for irregular packing
$Nu, b$	Nusselt number based on the particle diameter, and $(T_w - T_b)$ , Eq. (2)		<i>Subscripts</i>
$Q$	heat flux	b	bulk
$Pe$	Peclet number	e	equivalent/effective
$Re$	Reynolds number based on the particle diameter, Eq. (7)	er	effective radial
$T$	temperature	f	fluid
$T_{fw}$	extrapolated fluid temperature at the wall	p	packing
$u$	superficial velocity	t	tube
		w	wall

with a 160 cm long packed section. The duct was horizontally oriented and heated from the top wall only by a uniform heat flux, while the bottom and all the side walls were insulated. The heating element consisted of a resistance heater with an adjustable electric power input. The separation distance between the top and the bottom walls was maintained at 10 cm with the width being 40 cm. Air flow was introduced after a 10 cm long calming section into the packed section. The packing of polyvinyl chloride Raschig rings (hollow cylinders) with the equivalent diameters of 3.85 and 3.26 cm, and expanded polystyrene spheres with diameters of 4.8, 3.8 and 2.9 cm were used in the duct with the equivalent diameter,  $d_e = 21.7$  cm. The effective transport parameters were obtained from the radial air flow temperature measurements at four stations along the duct in the absence of a reaction. The experimental apparatus is given in detail elsewhere [15].

**3. Effective transfer parameters**

The average Nusselt number  $Nu$  is defined based on particle diameter and fluid thermal conductivity  $k_f$  ( $k_f = 0.027 \text{ W m}^{-1} \text{ K}^{-1}$  for air at  $35^\circ\text{C}$ ), and two sets of temperature differences as follows

$$Nu, a = \frac{Q d_p}{k_f (T_w - T_{fw})} \quad (1)$$

$$Nu, b = \frac{Q d_p}{k_f (T_w - T_b)} \quad (2)$$

where  $Q$  is the average heat flux on the heated top wall,  $T_{fw}$  is the extrapolated air temperature at the wall using the measured radial temperatures at a packed section which is chosen near the middle of axial length to ensure that the end effects are excluded.  $T_b$  is the bulk temperature of the air flow calculated from a third degree polynomial expression at a specified axial distance  $x$

$$T(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 \quad (3)$$

by the expression

$$T_b = \frac{\int_0^y T dy}{\int_0^y dy} = a_0 + \frac{a_1}{2} y + \frac{a_2}{3} y^2 + \frac{a_3}{4} y^3 \quad (4)$$

The parameters of the polynomial  $a_i$  are fitted to experimental radial temperature data including the extrapolated air temperature at the wall.  $Nu, a$  refers to the film at the wall of the duct and depends on the flow there. This implies that the heat transfer resistance in the immediate vicinity of the wall [1,8],  $Nu, b$  is a lumped parameter accounting various effects such as radial and axial dispersion of heat, lateral distribution of fluid velocity and of thermal conductivity [8]. The equivalent diameters for Raschig ring packing  $d_p$ , and the rectangular packed duct  $d_e$  are obtained as

$$d_e = \left[ 2.55K \frac{(WH)^2}{W+H} \right]^{1/3}; \quad d_p = \frac{6(1-\varepsilon)}{\Phi a_p} \quad (5)$$

where  $K$  is a constant that depends on  $W/H$  for a rectangular duct, and  $\Phi$  is the shape factor for an irregular packing [21].

The effective radial thermal conductivity is obtained from

$$-k_{er} \left( \frac{\partial T}{\partial y} \right)_{y=0} = h_w(T_w - T_{fw}) = Q \quad (6)$$

The radial temperature gradient at the wall is approximated from the polynomial expression given in Eq. (3).

#### 4. Results and discussion

Experiments were conducted with the heat flux range of 50 to 170 W/m<sup>2</sup>, and top wall temperature range of 30 to 45°C. All the heat transfer data is presented in terms of the Nusselt and the Reynolds numbers based on the diameter of the particle  $d_p$ . The Prandtl number is assumed to be constant. The Reynolds number based on  $d_p$  is given by

$$Re = \frac{u d_p}{\nu} \quad (7)$$

where  $u$  is the superficial velocity,  $\nu$  is the kinematic viscosity and  $k_f$  is the thermal conductivity of the fluid. The direct measurements of heat flux and the radial distribution of temperature of air flow have

been used in calculating the values of  $Nu$  and  $k_{er}/k_f$ . The experimental uncertainties due to air flow rate and temperature measurements in  $Nu$  and  $k_{er}/k_f$  are estimated to be in the range from 6 to 18%

The correlation equations for  $Nu$  and  $k_{er}/k_f$  obtained from the present measurements with the polyvinyl chloride Raschig rings are

$$Nu, a = 0.430 Re^{0.633} \quad (8)$$

$$Nu, b = 0.197 Re^{0.718} \quad (9)$$

$$\frac{k_{er}}{k_f} = 2.894 + 0.068 Re \quad (10)$$

for  $5.6 < d_e/d_p < 6.6$ ,  $200 < Re < 1200$ . The measurements with the expanded polystyrene spheres yield the following correlation equations

$$Nu, a = 0.217 Re^{0.756} \quad (11)$$

$$Nu, b = 0.047 Re^{0.927} \quad (12)$$

$$\frac{k_{er}}{k_f} = 10.432 + 0.0481 Re \quad (13)$$

for  $4.5 < d_e/d_p < 7.5$ ,  $200 < Re < 1450$ . The plots of experimental and the predicted values of  $Nu$  vs  $Re$  for Raschig rings and spheres are presented in Figs. 1 and

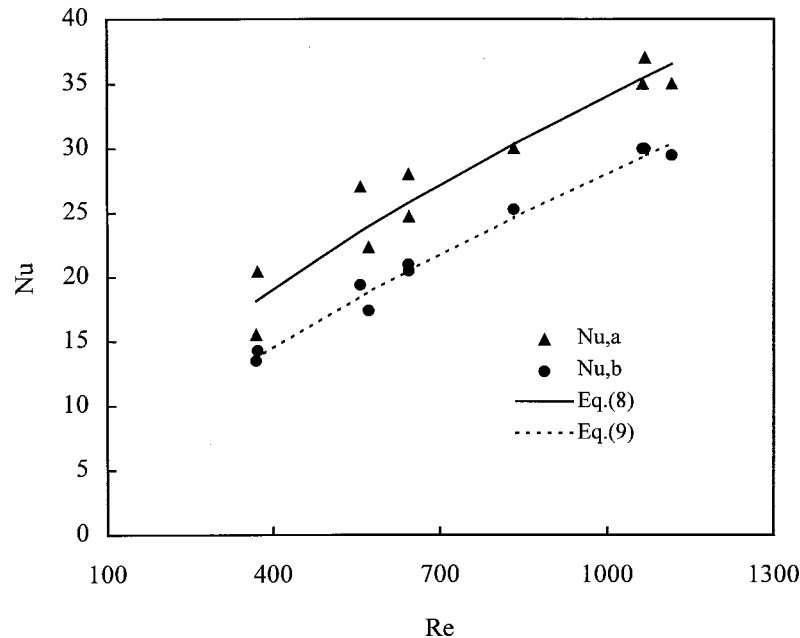


Fig. 1. Measured and predicted average Nusselt numbers  $Nu$  for rectangular duct packed with polyvinyl chloride Raschig rings (hollow cylinders) of  $5.6 < d_e/d_p < 6.6$ .

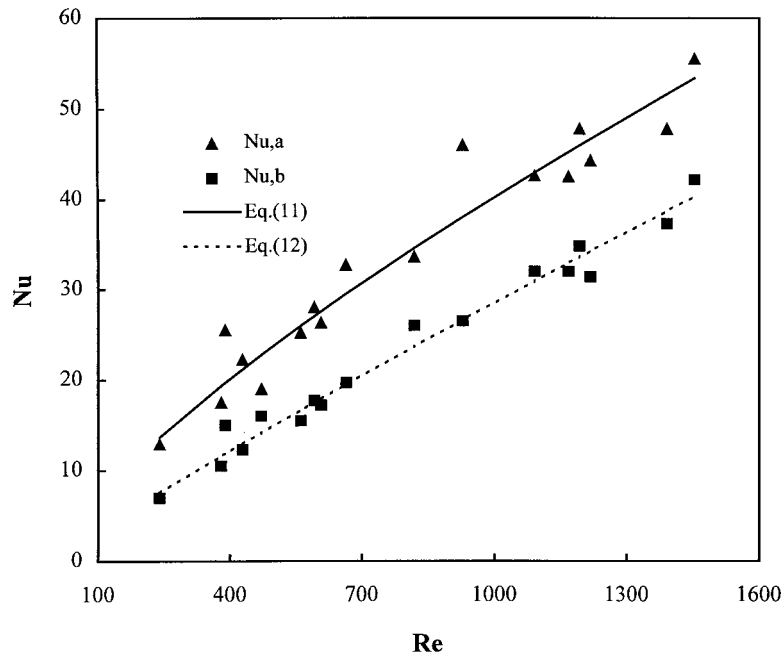


Fig. 2. Measured and predicted average Nusselt numbers,  $Nu$  for rectangular duct packed with expanded polystyrene (nonporous) spheres of  $4.5 < d_c/d_p < 7.5$ .

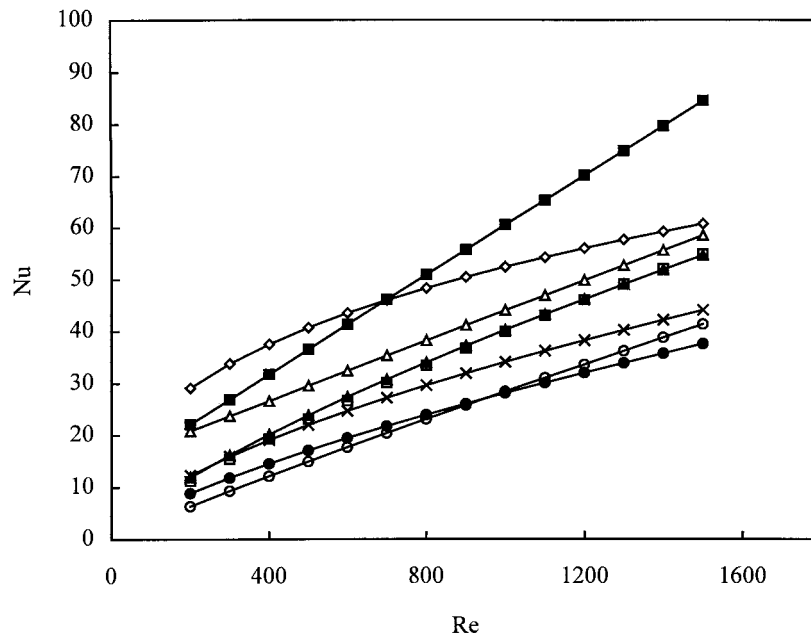


Fig. 3. Comparison of correlations for  $Nu$ .  $\diamond$ : Calderbank and Pogorski [13],  $Nu = 4.21Re^{0.365}$ ,  $14 < d_i/d_p < 28$ , celite cylinders,  $8 < d_i/d_p < 16$  alundum spheres.  $\blacksquare$ : DeWasch and Froment [12],  $Nu = 12.5 + 0.048Re$ ,  $d_i/d_p = 17$  cylinder.  $\triangle$ : Yagi and Kunii [11],  $Nu = 15 + 0.029Re$ , annular packed bed,  $3.9 < d_c/d_p < 51$ , glass beads; cylindrical bed,  $6 < d_i/d_p < 24$  celite balls, glass beads.  $\square$ : Li and Finlayson [14],  $Nu = 0.17Re^{0.790}$ ,  $3 < d_i/d_p < 5$ , celite spheres.  $\times$ : This study,  $Nu, a = 0.430Re^{0.633}$ , Eq. (8).  $\bullet$ : This study,  $Nu, b = 0.197Re^{0.718}$ , Eq. (9) [Eqs. (8) and (9) from rectangular packed bed with polyvinyl chloride Raschig rings,  $5.6 < d_c/d_p < 6.6$ ].  $\blacktriangle$ : This study,  $Nu, a = 0.217Re^{0.756}$ , Eq. (11).  $\circ$ : This study,  $Nu, b = 0.047Re^{0.927}$ , Eq. (12) [Eqs. (11) and (12) from rectangular packed duct with expanded polystyrene spheres,  $4.5 < d_c/d_p < 7.5$ ].

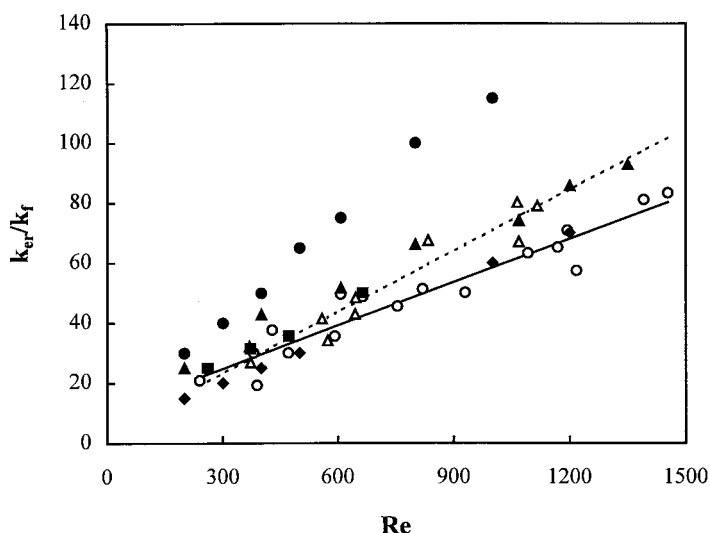


Fig. 4. Measured and predicted values of  $k_{cr}/k_f$ . ●: Dixon [5], ceramic hollow cylinders,  $d_i/d_p=5.1$ . ◆: Dixon [5], spheres,  $d_i/d_p=6.4$ . ■: Dixon [4], steel spheres,  $d_i/d_p=7.9$ . ▲: Freiwald and Paterson [10], ceramic sphere  $d_i/d_p=7.7$ . ○: This study, expanded polystyrene spheres,  $4.5 < d_e/d_p < 7.5$ . △: This study, polyvinyl chloride Raschig rings,  $5.6 < d_e/d_p < 6.6$ . —: This study, Eq. (13), expanded polystyrene spheres,  $4.5 < d_e/d_p < 7.5$ . - - -: This study, Eq. (10), polyvinyl chloride Raschig rings,  $5.6 < d_e/d_p < 6.6$ .

2, respectively. There is a considerable difference between the values of  $Nu$ ,  $a$  and  $Nu$ ,  $b$ , and the difference is relatively larger for spheres than that of Raschig rings. The correlation equations for  $Nu$  are compared with the various other correlation equations reported in the literature in Fig. 3. Eq. (11) and the correlation given by Li and Finlayson [14] overlap. It can be observed that the present results of  $Nu$ ,  $b$  indicate slightly lower values than those of the previous correlation equations. The equations shown in Fig. 3 are the results of the wide range of experimental conditions and show the discrepancies in the correlations. The interceptions of  $Nu$  at  $Re = 0$  are different from each other for various correlations, however the dependencies of  $Re$  as indicated by the slopes of the curves are fairly similar except the one given by DeWasch and Froment [12].

The predictions and experimental values of  $k_{cr}/k_f$  are shown in Fig. 4. Except the data of ceramic hollow cylinders with  $d_i/d_p=5.1$  by Dixon [5], the other data and our data are in a good agreement. The values of  $k_{cr}/k_f$  for Raschig ring packing are larger than those of spheres which are in line with Dixon [5].

#### Acknowledgements

The authors wish to acknowledge King Fahd University of Petroleum and Minerals for funding the research project CHE/HEATFLUX/173 that resulted in generating this study.

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